

## TESTING LORENTZ SYMMETRY IN SPACE

NEIL RUSSELL

*Physics Department,  
Northern Michigan University,  
1401 Presque Isle Avenue, Marquette, MI 49855, USA  
E-mail: nrussell@nmu.edu*

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Atomic clocks, masers, and other precision oscillators are likely to be placed on the International Space Station and other satellites in the future. These instruments will have the potential to measure Lorentz-violation coefficients, and in particular may provide access to parts of the Lorentz-violation coefficient space at levels not accessible with Earth-based experiments. The basic issues are outlined in this proceedings.

### 1. Lorentz-Violating Standard-Model Extension (SME)

The Standard-Model Extension (SME) is essentially the conventional Standard-Model lagrangian of particle physics plus all possible coordinate-independent Lorentz- and CPT-violating terms constructed from the conventional fields of particle physics.<sup>1,2</sup> The additional terms could arise in a more fundamental theory, for example string theory.<sup>3</sup> Since the symmetry-violating effects are known to be small, perturbative methods can be adopted to calculate the effects in any experimental context. Calculations or measurements for the SME in various systems include investigations of mesons,<sup>4</sup> neutrino oscillations,<sup>5</sup> spin-polarized matter,<sup>6</sup> hydrogen and antihydrogen,<sup>7</sup> Penning traps,<sup>8</sup> muons,<sup>9</sup> cosmological birefringence,<sup>10</sup> electromagnetic cavities,<sup>11</sup> electromagnetostatics,<sup>12</sup> and Čerenkov radiation.<sup>13</sup> Various other issues, including the SME in curved spacetime,<sup>14</sup> have been examined in the literature.<sup>15</sup>

An SME analysis of clock comparison experiments<sup>16</sup> provides a comprehensive framework for relating various tests.<sup>17</sup> These experiments search for signals that are due to rotations and accelerations of the laboratory relative to an inertial reference frame. It is therefore natural to consider

clock-comparison experiments performed in space since the laboratory motion offers various advantages. These proceedings provide an overview of the basic results of this analysis.<sup>18,19</sup>

## 2. General Clock-comparison Experiments

An atomic clock is a device that provides a stable transition frequency in a particular type of atomic system. For most atoms of interest, the total atomic angular momentum and its projection along the quantization axis are conserved to a high precision, so the quantum states can be labeled as  $|F, m_F\rangle$ . The shift in the energy levels due to the SME is found using a perturbation calculation giving

$$\begin{aligned} \delta E(F, m_F) = & \hat{m}_F \sum_w (\beta_w \tilde{b}_3^w + \delta_w \tilde{d}_3^w + \kappa_w \tilde{g}_d^w) \\ & + \tilde{m}_F \sum_w (\gamma_w \tilde{c}_q^w + \lambda_w \tilde{g}_q^w) \quad . \end{aligned} \quad (1)$$

The constants  $\hat{m}_F$  and  $\tilde{m}_F$  are ratios of Clebsch-Gordan coefficients given by

$$\hat{m}_F := \frac{m_F}{F} \quad , \quad \tilde{m}_F := \frac{3m_F^2 - F(F+1)}{3F^2 - F(F+1)} \quad . \quad (2)$$

In Eq. (1), the five tilde quantities are specific combinations of the coefficients for Lorentz violation within the SME. In the case of  $\tilde{d}_3^w$ , the definition is

$$\tilde{d}_3^w := m_w d_{03}^w + \frac{1}{2} m_w d_{30}^w - \frac{1}{2} H_{12}^w \quad . \quad (3)$$

Similar definitions apply for the remaining four tilde coefficients.<sup>16</sup> Noting that  $m_w$  is the mass of particle  $w$ , all five tilde coefficients have dimensions of mass. The index  $w$  is to be replaced with  $p$  for proton,  $e$  for electron, or  $n$  for neutron. The numerical subscripts refer to the laboratory-frame coordinate system, in which the third coordinate is the quantization axis by convention. Interestingly, these five tilde combinations are the only SME parameter combinations that can be bounded in clock-comparison experiments with ordinary matter. The aim of this work is to consider ways that atomic clock transition frequencies may be used to detect these tilde quantities. The five Greek-letter coefficients  $\beta_w, \gamma_w, \delta_w, \kappa_w, \lambda_w$  appearing in Eq. (1) are linear combinations of expectation values calculated for the state  $|F, F\rangle$  of particular operators in the nonrelativistic hamiltonian for

the particle  $w$ . For example, in the case of  $\delta_w$ , the expression is:

$$\delta_w := \frac{1}{m_w^2} \sum_{N=1}^{N_w} \langle [p_3 p_j \sigma^j]_{w,N} \rangle, \quad (4)$$

where  $p_j$  are the momentum operators and  $\sigma^j$  are the three Pauli matrices. These quantities are calculated for each particle of type  $w$  in a specific atom and the index  $N$  labels each of the  $N_w$  particles of that type; for example, in  $^{133}\text{Cs}$ ,  $N_p = N_e = 55$  and  $N_n = 78$ .

To calculate the values of  $\delta_w$  and the other similar coefficients would require a detailed understanding of the many-body nuclear physics. However, reasonable approximations can be made within specific nuclear models. Dimensional arguments indicate that  $\beta_w$  is of order unity, and the other quantities are suppressed by factors of about  $K_p \approx K_n \simeq 10^{-2}$  and  $K_e \simeq 10^{-5}$ .

The frequency output  $f(B_3)$  of a typical atomic clock is determined by the difference between two energy levels and in general depends on the magnetic field projected on the quantization axis,  $B_3$ . Including the Lorentz-violating effects  $\delta\omega$ , the output frequency  $\omega$  is expressed as

$$\omega = f(B_3) + \delta\omega. \quad (5)$$

The transition frequency  $\omega$  is affected by both of the levels in the transition  $(F, m_F) \rightarrow (F', m'_F)$ , so  $\delta\omega$  is determined from

$$\delta\omega = \delta E(F, m_F) - \delta E(F', m'_F). \quad (6)$$

### 3. Standard Inertial Reference Frame

The Lorentz-violating effects in equation (1) are contained in the SME tilde quantities, which are tensors under observer transformations. Thus, their components in one inertial reference frame are related to those in another by the corresponding rotation or boost between observers. However, unlike the energy-momentum tensor, for example, they are not integrated from controllable experimental source configurations. They are instead fixed in space. In conventional physics, results are independent of the orientation or velocity of the laboratory, but this is no longer true since the interaction of the experiment with this fixed Lorentz-violating background introduces time-dependent effects. A measurement of  $\tilde{d}_3^w$ , for example, is time-dependent since the third component in the laboratory frame is changing its orientation as the Earth rotates.

The time dependence is determined by the laboratory motion relative to a standard reference frame. By convention, this frame is centered on the Sun with  $Z$  axis parallel to the rotation axis of the Earth, and with  $X$  axis pointing at the vernal equinox on the celestial sphere. The time  $T$  is measured from the vernal equinox in the year 2000. Measurement of the SME coefficients in the standard frame is done using the laboratory trajectory through a sequence of linear transformations. For the case of a satellite, the motion is a combination of the circular motion of the Earth around the Sun and the circular motion of the satellite around the Earth. As an example of one of the laboratory-frame quantities expressed in terms of the inertial frame, the expression for  $\tilde{d}_3$  is

$$\begin{aligned} \tilde{d}_3 = & \cos \omega_s T_s \left\{ \left[ \tilde{d}_X (-\sin \alpha \cos \zeta) + \tilde{d}_Y (\cos \alpha \cos \zeta) + \tilde{d}_Z (\sin \zeta) \right] \right. \\ & \left. + \beta_{\oplus} [\text{seasonal Sun-frame tilde terms}] \right\} \\ & + \sin \omega_s T_s \left\{ \left[ \tilde{d}_X (-\cos \alpha) + \tilde{d}_Y (-\sin \alpha) \right] \right. \\ & \left. + \beta_{\oplus} [\text{seasonal Sun-frame tilde terms}] \right\} \\ & + \cos 2\omega_s T_s \left\{ \beta_s [\text{constant Sun-frame tilde terms}] \right\} \\ & + \sin 2\omega_s T_s \left\{ \beta_s [\text{constant Sun-frame tilde terms}] \right\} \\ & + \left\{ \beta_s [\text{constant Sun-frame tilde terms}] \right\}. \end{aligned} \quad (7)$$

Here, the  $z$  or 3 direction in the lab is oriented along the velocity vector of the satellite relative to the Earth, while the  $x$  direction points towards the center of the Earth. The satellite time  $T_s$  is related to the Sun-based time by  $T = T_s + T_0$ , where  $T = T_0$  is the time of a selected ascending node of the satellite. Other satellite orbital elements in the expression are the right ascension  $\alpha$  of the ascending node and the inclination  $\zeta$  between the orbital axis and the Earth's axis. For the International Space Station,  $\omega_s \approx 2\pi/92\text{min}$  and  $\beta_s \approx 3 \times 10^{-5}$  are the orbital frequency and speed relative to the Earth. The speed of the Earth is  $\beta_{\oplus} \approx 1.0 \times 10^{-4}$ , and the seasonal terms refer to cyclic variations with angular frequency  $2\pi/(\text{one sidereal year})$ .

In Eq. (7), only the sun-frame tilde components  $\tilde{d}_X$ ,  $\tilde{d}_Y$ , and  $\tilde{d}_Z$  appear explicitly. Others appear in the seasonal and constant Sun-frame expressions, which are given in full and in tabular form in Ref. 19. Both single and double frequencies appear in the expressions, and can be understood as arising from single- and double-index coefficients in the SME. An advan-

tage of using a satellite is the relatively high frequency  $\omega_s$  which reduces the limitation of clock stability over time. Use of a turntable in a ground-based laboratory, as is being done in some experiments, offers a similar stability payoff although the velocity factor  $\beta_s$  is reduced 16-fold to the value  $\beta_L \approx 1.6 \times 10^{-6}$ .

The coefficients that a particular clock-comparison experiment could detect in principle depend on the atoms of the clock and the transition used. An analysis has been done for rubidium clocks, cesium clocks, and hydrogen masers.<sup>19</sup> Similar techniques can be applied to other systems.

#### 4. Discussion

There are 120 coefficients that in principle clock-comparison experiments can detect at leading order, consisting of 40 for each of the three basic subatomic particles. About half of these coefficients are suppressed by a factor of  $\beta_s$ , indicating that detection of these coefficients may be enhanced in a satellite moving at high  $\beta_s$ . A number of coefficients have been probed with earth-based experiments, even though the lab speed relative to the Earth is an order of magnitude less than in orbit. If experiments were done today with cesium and rubidium atomic clocks in space, several dozen unmeasured coefficients would be accessed. Others would be accessible with different clocks, and in principle, all 120 coefficients are accessible from space-based clock-comparison experiments.

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